CONSERVATIVE GRIDS FOR 2D ELECTROMAGNETIC PROBLEMS WITH ARBITRARY DISTRIBUTION AND VALUES OF PERMITTIVITY AND PERMEABILITY

Constantin Klimov, Boris Sestroretsky

Two-dimensional equivalent circuits of elementary volumes of space are built in the paper on the base of lumped and distributed elements. They allow to form conservative grids for two-dimensional simulation of problems with arbitrary values and distribution of permittivity and permeability. Numerical algorithms constructed on the base of the suggested grids are unconditionally stable for materials with positive, negative and zero permittivity and permeability.

INTRODUCTION

To solve a problem of analysis of a system, it is necessary to build a space element operator, equivalent to the Maxwell's equations. Descriptions of building of such operators for planar structures can be found in [2], [5], [9], [10]. However it is necessary to make some additions to those operators or circuits, which would make it possible to get a computationally stable procedure for media with dielectric constants $\varepsilon < \varepsilon_0$ (ε_0 is dielectric constant of vacuum) and with magnetic permeability $\mu < \mu_0$ (μ_0 is magnetic permeability of vacuum). A physical meaning of the space element operator is self-descriptive in the frequency domain (it is called RLC equivalent circuit and composed of lumped elements). Thus, we are going to build a lumped element equivalent circuit first and then derive $R\tau$ circuits composed of transmission line segments and stubs, which are convenient for the time domain simulations.

RLC EQUIVALENT CIRCUIT OF AN ELEMENT OF SPACE

The Maxwell's equations in the differential form can be written as

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}^{e} , \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{j}^{m} \tag{1}$$

For the isotropic dielectric material the constitutive equations are

$$\vec{D} = \varepsilon_a \vec{E} \quad , \qquad \vec{B} = \mu_a \vec{H} \tag{2}$$

For the time-harmonic electromagnetic fields with an angular frequency ω we have

$$\frac{\partial \vec{D}}{\partial t} = i\omega \vec{D} ; \qquad \frac{\partial \vec{B}}{\partial t} = i\omega \vec{B} \tag{3}$$

Thus, for a two-dimensional problem (there is no variations of geometry and fields along one of the coordinate axis, along the z-axis for instance) of analysis of waves with H polarization ($H_z = 0; E_x = E_y = 0$) in a region without sources ($\vec{j}^{e} = 0$ и $\vec{j}^{m} = 0$) and taking into account equations (2) and (3), the Maxwell's equations (1) can be written in the Cartesian coordinate system as

$$\frac{\partial E_z}{\partial y} = -i\omega\mu_a H_x ; \quad -\frac{\partial E_z}{\partial x} = -i\omega\mu_a H_y ; \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon_a E_z \tag{4}$$

The system of the equations (4) can be further reduced to the Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right) \cdot E_z = 0 \tag{5}$$

where $k^2 = \omega^2 \varepsilon_a \mu_a$, k is wave number.

The system of equations (4) can be written down in a finite differences form

$$\frac{\Delta E_z}{\Delta y} = -i\omega\mu_a H_x, \quad -\frac{\Delta E_z}{\Delta x} = -i\omega\mu_a H_y, \quad \frac{\Delta H_y}{\Delta x} - \frac{\Delta H_x}{\Delta y} = i\omega\varepsilon_a E_z \qquad (6)$$

The system (6) can be solved directly, as it is usually done in the finite difference method. An alternative is to establish correspondence between equations (6) and an equivalent RLC-circuit shown in Fig. 1 like it is done in [1]-[6]. The circuit is a simple extension of an equivalent circuit of a transmission line segment. Voltages U on the capacitors $\frac{C}{4}$ correspond to the intensity of an electrical field E_z, and currents I_x and I_y through the inductors $2L_x$ and $2L_y$ correspond to the intensity of magnetic field components H_y and H_x respectively. The correspondence is following:

$$U = E_z \cdot \Delta_z, \qquad I_x = H_y \cdot \Delta_y, \qquad I_y = H_x \cdot \Delta_x, \qquad (7)$$

where Δ_x, Δ_y, Δ_z - the geometrical sizes of an element of space (rectangular parallelepiped) along the x, y and z coordinate axes respectively.

The capacitances and inductances of the elements of the equivalent circuit are defined through the element sizes and electrical parameters of the element medium according to [9], [10], [16], [17]:

$$C = \varepsilon_a \frac{\Delta_x \cdot \Delta_y}{\Delta_z}, \quad L_x = \mu_a \frac{\Delta_x \cdot \Delta_z}{\Delta_y}, \quad L_y = \mu_a \frac{\Delta_y \cdot \Delta_z}{\Delta_x}, \qquad (8)$$

where ε_a - absolute permittivity, μ_a - absolute permeability of the medium.

A complete geometry of a problem is assembled from the equivalent circuits of the elementary volumes like it is shown in Fig. 2. In other words, at first, we decomposed a complex problem into elementary volumes represented by the similar equivalent circuits. Then, an equivalent circuit of the whole problem is recomposed from the circuits of the elementary volumes [7]. Instead of the electromagnetic problem, we obtain a network-theory problem of the circuit or multi-port simulation [12]. Boundary conditions can be simulated as appropriate terminations of some ports of the circuit. Short circuit terminations, for instance, represent the ideal metal boundary conditions and open circuit terminations correspond to the magnetic wall conditions. The outlined procedure is referred as the building of the impedance model of electromagnetic problem. It allows proceeding from the language of differential equations or from the formal mathematical language to the description of a problem in the language of impedance networks or circuits. In contrast to the formal mathematical description, the circuit-theory description allows to operate with the model in more natural way.

The described impedance model allows to solve electromagnetic problems for media with $\varepsilon_a > \varepsilon_0$ or $\mu_a > \mu_0$. A formal introduction of the equivalent circuits with the negative capacitances C to simulate media with $\varepsilon_a < 0$ (or negative L_x and L_y for $\mu_a < 0$) means actually connection of a generator in the equivalent circuit that leads to numerical instability. To obtain a numerically stable procedure for the electromagnetic simulation of medium with $\varepsilon_a \leq 0$ and $\mu_a \leq 0$, it is necessary to modify the circuit of the element of space shown in Fig. 1. It requires some modifications of the differential operator (4).

Let us use a representation of the electromagnetic space as described in [12], [13], [17]. At such approach, we to the space, blank by substance, put in conformity multi-port (fig. 2), formed recomposition of elementary volumes of space (fig. 3).

The nominal values of elements of such equivalent circuit are defined by the following from (8) expressions:

$$C_0 = \varepsilon_0 \frac{\Delta_x \cdot \Delta_y}{\Delta_z}; \quad L_{x0} = \mu_0 \frac{\Delta_x \cdot \Delta_z}{\Delta_y}; \; L_{y0} = \mu_0 \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \tag{9}$$

, where ε_0 - permittivity of vacuum, μ_0 - permeability of vacuum.

The substance is simulated by connection to the received circuit two-ports. I.e. influence of substance - locally, and all interaction carries out through vacuum (fig. 2). Such model name by model information multiport [12].

If in a considered element of space absolute the permittivity is distinct from permittivity of vacuum ε_0, to the circuit of an element of space (fig. 3) we connect two-ports $Y/4$ (fig. 4).

The parameters connected two-ports should be such, that on frequency of the analysis ω of the circuit represented on a fig. 3 and fig. 1 were equivalent.

Besides not we shall while enter into a grid of elements absorbing and generating energy. Let our grid will be conservative. It means, that two-ports $Y/4$ will be either inductance, or capacity. I.e. it is necessary to consider two cases.

First corresponds $\varepsilon_a > \varepsilon_0$:

$$\varepsilon_a = \varepsilon_0 + \varepsilon^+; \qquad C = C_0 + C^+; \qquad C^+ = \varepsilon^+ \frac{\Delta_x \cdot \Delta_y}{\Delta_z} \tag{10}$$

Thus the substance increases C_0 capacity of an element of space on C^+. The equivalent circuit of an element of space of a fig. 4 will be transformed to the circuit of a fig. 5.

The second case corresponds $\varepsilon_a < \varepsilon_0$:

$$\varepsilon_a = \varepsilon_0 - \varepsilon^-; \qquad C = C_0 - C^-; \qquad C^- = \varepsilon^- \frac{\Delta_x \cdot \Delta_y}{\Delta_z}; \; L^- = \frac{1}{\omega^2 C^-} \tag{11}$$

Thus the substance reduces C_0 capacity of an element of space on C^-. This reduction of capacity is equivalent to entering of inductance L^-. I.e. at $\varepsilon_a < \varepsilon_0$ substance brings in additional inductance, reducing, thus, capacity of vacuum C_0. Thus, the equivalent circuit of an element of space of a fig. 4 will be transformed to the circuit of a fig. 6 [18].

Such treatment gives the evident explanatory of special cases $\varepsilon_a = 0$ and $\varepsilon_a < 0$.

At $\varepsilon_a = 0$ for parallel contours formed inductance's $4 \cdot L^-$ and capacities $C_0 / 4$, the parallel resonance is observed:

$$L^- = \frac{1}{\omega^2 C_0} \tag{12}$$

That is the parallel contour is replaced with break.

At $\varepsilon_a < 0$ the inductive component of total conductivity of a contour $\frac{1}{\omega \cdot 4 \cdot L^-}$ exceeds a capacitor component $\omega \cdot C_0 / 4$, i.e.:

$$L^- < \frac{1}{\omega^2 C_0} \tag{13}$$

In both considered above cases - (12) and (13), the conductivity of a contour either is equal to zero, or has inductive character. Therefore waves in a grid will not be distributed. The case $\varepsilon_a = 0$ reminds a situation in rectangular waveguide, when the frequency of a stimulating signal is equal to critical frequency for a wave of the lowest type. Happen $\varepsilon_a < 0$ - when frequency of a signal less critical frequency of a wave of the lowest type.

That in our impedance grid there were propagating waves, it is necessary to supply performance of a condition $0 < \varepsilon_a$. In nominal values of the equivalent circuit of a fig. 6 it means, that:

$$L^- > \frac{1}{\omega^2 C_0} \tag{14}$$

It is similarly possible to simulate filling of space by substance with permeability μ_a distinct from permeability of vacuum μ_0.

Thus the substance is simulated by connection to the circuit (fig. 3) two-ports $2 \cdot Z_x$ and $2 \cdot Z_y$ consistently to inductance $2 \cdot L_{x0}$ and $2 \cdot L_{y0}$, so that on frequency of the analysis ω the circuit represented on a fig. 7 and in a fig. 1 were equivalent.

As our grid is conservative, two-ports $2 \cdot Z_x$ and $2 \cdot Z_y$ either is inductance or capacity. Therefore it is necessary to consider two cases [21].

First corresponds $\mu_a > \mu_0$:

$$\mu_a = \mu_0 + \mu^+; \qquad L_x = L_{x0} + L_x^+; \qquad L_y = L_{y0} + L_y^+;$$

$$L_x^+ = \mu^+ \frac{\Delta_x \cdot \Delta_z}{\Delta_y}; L_y^+ = \mu^+ \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \tag{15}$$

Thus the substance increases L_{x0} and L_{y0} - inductance of an element of space on L_x^+ and L_y^+ on accordingly. The equivalent circuit of an element of space of a fig. 7 will be transformed to the circuit of a fig. 8.

The second case corresponds $\mu_a < \mu_0$:

$$\mu_a = \mu_0 - \mu^-; \qquad L_x = L_{x0} - L_x^-; \qquad L_y = L_{y0} - L_y^-;$$

$$L_x^- = \mu^- \frac{\Delta_x \cdot \Delta_z}{\Delta_y} \quad ; L_y^- = \mu^- \frac{\Delta_y \cdot \Delta_z}{\Delta_x} \tag{16}$$

$$C_x^- = \frac{1}{\omega^2 L_x^-}; \qquad C_y^- = \frac{1}{\omega^2 L_y^-} \tag{17}$$

Thus the substance reduces L_{x0} and L_{y0} - inductance of an element of space on L_x^- and on L_y^- accordingly. This reduction inductance is equivalent to entering of capacities C_x^- and C_y^- (see fig. 9). I.e. at $\mu_a < \mu_0$ substance brings in additional capacities, reducing, thus, of inductance of vacuum L_{x0} and L_{y0}. Thus, the equivalent circuit of an element of space of a fig. 7 will be transformed to the circuit of a fig. 9.

Let's consider special cases $\mu_a = 0$ and $\mu_a < 0$.

At $\mu_a = 0$ for consecutive contours formed inductive $2L_{x0}$ and capacities $C_x^- / 2$, and for contours consisting from $2L_{y0}$ and $C_y^- / 2$, the consecutive resonance is observed:

$$C_x^- = \frac{1}{\omega^2 L_{x0}}; \qquad C_y^- = \frac{1}{\omega^2 L_{y0}} \tag{18}$$

And the oscillatory contour is replaced with short circuit.

At $\mu_a < 0$ the capacitor component of total resistance of a contour along an axis x $\frac{1}{\omega \cdot 2 \cdot C_x^-}$ (and for a contour along an axis y $\frac{1}{\omega \cdot 2 \cdot C_y^-}$) exceeds an inductive component $\omega \cdot 2 \cdot L_{x0}$ ($\omega \cdot 2 \cdot L_{y0}$), i.e.:

$$C_x^- < \frac{1}{\omega^2 L_{x0}}; \qquad C_y^- < \frac{1}{\omega^2 L_{y0}} \tag{19}$$

In both considered above cases (18) and (19), the resistance of contours or is equal to zero, or has capacitor character. Therefore waves in a grid in these cases will not be propagated.

That in an impedance grid there were propagating waves, it is necessary to ensure performance of a condition $0 < \mu_a$. In nominal values of the equivalent circuit of a fig. 9 it means, that:

$$C_x^- > \frac{1}{\omega^2 L_{x0}}; \qquad C_y^- > \frac{1}{\omega^2 L_{y0}} \tag{20}$$

For modeling substance in the equivalent circuit of elementary area of space at $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ it is necessary to use combinations of the circuits of a fig. 5, fig. 6, fig. 8 and rice 9, depending values ε_a and μ_a (see fig. 10 - 13).

The nominal values of elements for the circuits represented in a fig. 10 - 13 are calculated with the help of expressions (10), (11), (15) and (16).

Thus, the conservative RLC-circuits of elementary volume of space for $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ 2D task of H polarization can be constructed. At assemblage of the given circuits the received grid of analyzed area too will be conservative, that means computing stability of procedure of the electromagnetic analysis.

REMARK: UPDATING OF THE INITIAL DIFFERENTIAL EQUATIONS

The constructed above impedance circuits of elementary volume of space strictly correspond to the differential operator (5) only at $\varepsilon_a > \varepsilon_0$ and $\mu_a > \mu_0$. As actually having replaced capacity (for $\varepsilon_a < \varepsilon_0$) and inductance (for $\mu_a < \mu_0$) according to parallel and consecutive oscillatory contours, we have increased number of degrees of freedom of analyzed system, having raised the order of the differential operator for preservation of conservatism of system. Let's explain the below given remark.

The initial equations (1) and (2) can be written down in the following kind:

$$\nabla \times \vec{H} = \frac{\partial \vec{E}}{\partial t}\varepsilon_a \quad ; \qquad \nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t}\mu_a \tag{21}$$

In a symbolical kind [25], [26]

$$\nabla \times \vec{H} = pC_e \vec{E} \quad ; \qquad \nabla \times \vec{E} = -pL_m \vec{H} \tag{22}$$

where $p = \frac{\partial}{\partial t}$ - the symbolical form of record of the operator of differentiation on time, $C_e = \varepsilon_a$ - specific equivalent electrical capacity, $L_m = \mu_a$ - specific equivalent magnetic inductance.

Use of model information multiport corresponds to record of the equations (22) in the following kind:

$$\nabla \times \vec{H} = (pC_{e0} + pC_e^+)\vec{E} \text{ for } \quad \varepsilon_a = \varepsilon_0 + \varepsilon^+ \qquad (\varepsilon_a > \varepsilon_0), \quad (23)$$

$$\nabla \times \vec{H} = (pC_{e0} + \frac{1}{pL_e^-})\vec{E} \quad \text{ for } \quad \varepsilon_a = \varepsilon_0 - \varepsilon^- \qquad (\varepsilon_a < \varepsilon_0), \quad (24)$$

$$\nabla \times \vec{E} = -(pL_{e0} + pL_m^+)\vec{H} \quad \text{ for } \quad \mu_a = \mu_0 + \mu^+ \qquad (\mu_a > \mu_0), \quad (25)$$

$$\nabla \times \vec{E} = -(pL_{e0} + \frac{1}{pC_m^-})\vec{H} \quad \text{ for } \quad \mu_a = \mu_0 - \mu^- \qquad (\mu_a < \mu_0), \quad (26)$$

where $p = \frac{\partial}{\partial t}$ - the symbolical form of record of the operator of differentiation on time, $\frac{1}{p} = \int \partial t$ - the symbolical form of record of the operator of integration on time, $C_{e0} = \varepsilon_0$ - specific electrical capacity of vacuum; $L_{e0} = \mu_0$ - specific magnetic inductance of vacuum; $C_e^+ = \varepsilon^+$ - specific electrical capacity of substance increasing capacity of vacuum; $L_m^+ = \mu^+$ - specific magnetic inductance of substance increasing inductance of vacuum; $C_m^- = -\frac{1}{p^2 \mu^-}$ - specific magnetic capacity of substance reducing inductance of vacuum; $L_e^- = -\frac{1}{p^2 \varepsilon^-}$ - specific electrical inductance of substance reducing capacity of vacuum. Apparently, for the description of properties of substances at $\varepsilon_a < \varepsilon_0$ ($\mu_a < \mu_0$) it would be more convenient to use not ε^- (μ^-), and sizes L_e^- (C_m^-).

The equations (23) - (26) correspond to the equivalent circuits of an element of space represented on a fig. 10 - 13.

It is possible to unit expressions (23) - (26) and to take into account specific electrical Y_e and specific magnetic losses R_m. Then initial equations (1), apparently, is possible to write down as:

$$\nabla \times \vec{H} = C_{e0}\frac{\partial \vec{E}}{\partial t} + C_e^+ \frac{\partial \vec{E}}{\partial t} + \frac{1}{L_e^-}\int \vec{E}\cdot \partial t + Y_e \vec{E} + \vec{j}^e \tag{27}$$

$$\nabla \times \vec{E} = -L_{m0}\frac{\partial \vec{H}}{\partial t} - L_m^+ \frac{\partial \vec{H}}{\partial t} - \frac{1}{C_m^-}\int \vec{H}\cdot \partial t - R_m \vec{H} - \vec{j}^m \tag{28}$$

Basically substance, probably, can behave by even more complex image, then for the description of properties of such substances, apparently, it will be necessary to increase number of degrees of freedom describing substance of model, that means additional increase about system of the differential equations (27) and (28)

THE EQUIVALENT $R\tau$-CIRCUIT OF AN ELEMENT OF SPACE

Let's proceed now from the considered above equivalent impedance RLC-circuits consisting of concentrated elements, to the $R\tau$-circuits [11], [17]. The $R\tau$-circuits of elementary volume of space consist of transmission lines lost-free. Let's consider further equivalent circuit of an element of space represented on a fig. 1 under condition of equality spatial discrete along axes x and y, i.e. we shall analyze a square grid:

$$\Delta_x = \Delta_y = \Delta \tag{25}$$

As before we shall use model information multi-port.

At performance of a condition (25) inductance's L_{x0} and L_{y0} also become equal, we shall designate them L_0, and the circuit (fig. 3) vacuum transform to the $R\tau$-circuit how is shown in a fig. 14 [11], [17].

On the circuit of a fig. 14 times of a delay of a signal in transmission lines are designated τ_0, wave admittance of lines - $Y_0 / 2$. Taking into account (9) and (25), it is possible to write down the following expressions for parameters of lines [17]:

$$\tau_0 = \Delta\sqrt{\frac{\varepsilon_0 \mu_0}{2}}; \qquad Y_0 = \frac{\Delta}{\Delta_z} \cdot \sqrt{\frac{\varepsilon_0}{2 \cdot \mu_0}} \tag{26}$$

As it is visible from (26), for planar (2D) of a task, the time of a delay of a signal in lines τ_0 corresponds to local speed of distribution of signals between two next units of a grid of speed in $\sqrt{2}$ time of the greater speed of light $c = 1/\sqrt{\varepsilon_0 \mu_0}$. The local wave conductivity of lines, from which consists a grid, also in $\sqrt{2}$ differs from meaning of wave conductivity of vacuum for flat waves.

If in a considered element of space absolute permittivity is distinct from permittivity of vacuum ε_0, to the circuit of an element of space (fig. 14) we connect two-port $Y/4$ (fig. 15), is similar to the circuit (fig. 4).

The parameters connected two-port should be such, that on frequency of the analysis ω of the circuit represented on a fig. 15 and fig. 4 were equivalent. Similarly to inclusion dielectric for the RLC-circuits, we consider for the $R\tau$-circuits 2 cases, which will correspond to conservative grids. In the first case $Y/4$ - capacity, in second - inductance.

The first variant corresponds $\varepsilon_a > \varepsilon_0$ (see fig. 5 and expression (10)). Thus, as shown in a fig. 16, we can simulate two-port $Y/4$ (capacity $C^+/4$) by open stubs of length appropriate to time of a delay of a signal in a line $\tau_0/2$ and wave admittance $Y_5/4$. For equivalence of the circuits represented in a fig. 5 and fig. 16 on frequencies ω, it is necessary to equate of conductivity of a stub with wave admittance $Y_5/4$ and conductivity of capacity $C^+/4$ [8], [17]:

$$\frac{Y_5}{4} \cdot tg\left(\frac{\omega\tau_0}{2}\right) = \frac{\omega C^+}{4} \tag{27}$$

Thus for a stub of length $\tau_0/2$ at $\omega\tau_0 << 1$ (27) with the account (10), (25) and (26) gets a kind:

$$\varepsilon_a = \varepsilon_0 + \varepsilon^+; \qquad y_5 = \frac{Y_5}{Y_0} = 4 \cdot \frac{\varepsilon^+}{\varepsilon_0}; \tag{28}$$

Where y_5 - normalized concerning local wave conductivity of vacuum Y_0 admittance of a stub simulating dielectric.

In this case there is some freedom in a choice of lengths of loops. To each chosen length of a loop there will correspond the meaning a admittance $Y_5/4$, at which will be satisfied condition equivalence of the circuits of a fig. 15 and fig. 3. Length appropriate to time of a delay of a signal in a line $\tau_0/2$, was chosen for reception of two-parametrical algorithm [17].

The second case corresponds $\varepsilon_a < \varepsilon_0$: (see fig. 6 and expression (11)). Thus, as shown in a fig. 17, we can simulate two-port $Y/4$ (inductance $4 \cdot L^-$) by stubs of short circuit of length appropriate to time of a delay of a signal in a line $\tau_0/2$ and a wave admittance $Y_5/4$. For equivalence of the circuits represented in a fig. 6 and fig. 17 on frequencies ω, it is necessary to equate of conductivity of a short stub with wave admittance $Y_5/4$ and conductivity of inductance $4 \cdot L^-$ [8], [17]:

$$\frac{Y_5}{4} \cdot ctg\left(\frac{\omega\tau_0}{2}\right) = \frac{1}{4\omega L^-} \tag{29}$$

Thus for a stub of length $\tau_0 / 2$ at $\omega\tau_0 << 1$ (29) with the account (11), (25) and (26) gets a kind:

$$\varepsilon_a = \varepsilon_0 - \varepsilon^- ; \qquad y_5 = \frac{Y_5}{Y_0} = (\omega\tau_0)^2 \cdot \frac{\varepsilon^-}{\varepsilon_0} ; \tag{30}$$

Where y_5 - normalized concerning local wave conductivity of vacuum Y_0 admittance of a short stub simulating dielectric with $\varepsilon_a < \varepsilon_0$.

Let's consider two special cases: $\varepsilon_a = 0$ and $\varepsilon_a < 0$.

At $\varepsilon_a = 0$, the normalized conductivity short stub y_5 defined following expression:

$$y_5 = (\omega\tau_0)^2 \tag{31}$$

At $\varepsilon_a < 0$:

$$y_5 > (\omega\tau_0)^2 \tag{32}$$

In both considered above cases - (31) and (32), appropriate (12) and (13), the waves in a grid will not be propagated.

That in an impedance grid at short stub (see fig. 17) there were propagating waves, it is necessary to ensure performance of a condition $0 < \varepsilon_a$. It means, that:

$$y_5 < (\omega\tau_0)^2 \tag{33}$$

It is similarly possible simulate filling of space by substance with permeability μ_a distinct from permeability of vacuum μ_0.

Thus the substance is simulated by consecutive connection to the circuit (fig. 14) two-ports $2 \cdot Z$ how is shown in a fig. 18 [14], [15], [17].

The parameters connected two-ports should be such, that on frequency of the analysis ω of the circuit represented on a fig. 18 and fig. 7 were equivalent.

Similarly to inclusion magnetic for the RLC -circuits, we shall consider for the $R\tau$ -circuits 2 cases, which will correspond to conservative grids. In the first case two-port $2 \cdot Z$ is inductance, in second - $2 \cdot Z$ is capacity.

The first variant corresponds $\mu_a > \mu_0$: (see fig. 8 and expression (15)). Thus, as shown in a fig. 19, we can simulate two-ports $2 \cdot Z$ (inductance $L^+ = L_x^+ = L_y^+$) by short stubs of length appropriate to time of a delay of a signal in a line $\tau_0 / 2$ and a wave impedance $2 \cdot Z_3$. For equivalence of the circuits represented in a fig. 8 and fig. 19 on frequencies ω, it is necessary to equate resistance of a stub with a wave impedance $2 \cdot Z_3$ and resistance to inductance $2 \cdot L^+$ [8], [17]:

$$2Z_3 \cdot tg\left(\frac{\omega\tau_0}{2}\right) = 2\omega L^+ \tag{34}$$

Thus for a stub of length $\tau_0 / 2$ at $\omega\tau_0 << 1$ (34) with the account (15), (25) and (26) gets a kind:

$$\mu_a = \mu_0 + \mu^+ ; \quad z_3 = \frac{Z_3}{Z_0} = 2 \cdot \frac{\mu^+}{\mu_0} ; \tag{35}$$

Where z_3 - normalized concerning a local impedance of vacuum $Z_0 = 1/Y_0$ wave resistance short stub simulating magnetic with $\mu_a > \mu_0$.

The second case corresponds $\mu_a < \mu_0$: (see fig. 9 and expression (16), (17)). Thus, as shown in a fig. 20, we can simulate capacity $C^- / 2$ by open stubs. The lengths of these open stubs correspond to time of a delay of a signal in a transmission line $\tau_0 / 2$ and have wave impedance $2 \cdot Z_3$. For equivalence of the circuits represented in a fig. 9 and fig. 20 on frequencies ω, it is necessary to equate resistance of a open stub with a wave impedance $2 \cdot Z_3$ and resistance to capacity $C^- / 2$ [8], [17]:

$$2Z_3 \cdot ctg\left(\frac{\omega\tau_0}{2}\right) = \frac{2}{\omega C^-} \tag{36}$$

Thus for a open stub of length $\tau_0 / 2$ at $\omega\tau_0 << 1$ (36) with the account (16), (17), (25) and (26) gets a kind:

$$\mu_a = \mu_0 - \mu^- ; \qquad z_3 = \frac{Z_3}{Z_0} = 2 \cdot (\omega\tau_0)^2 \cdot \frac{\mu^-}{\mu_0} ; \tag{37}$$

Where z_3 - normalized concerning a local impedance of vacuum $Z_0 = 1/Y_0$ wave resistance open stub simulating magnetic with $\mu_a < \mu_0$.

Let's consider two special cases: $\mu_a = 0$ and $\mu_a < 0$.

At $\mu_a = 0$, z_3 - the normalized wave resistance of a open stub defined following expression:

$$z_3 = 2 \cdot (\omega\tau_0)^2 \tag{38}$$

At $\mu_a < 0$:

$$z_3 > 2 \cdot (\omega\tau_0)^2 \tag{39}$$

In both considered above cases - (38) and (39), appropriate (18) and (19), the waves in a grid will not be propagated.

That in an impedance grid at a open stub (see fig. 20) there were propagating waves, it is necessary to supply performance of a condition $0 < \mu_a$. It means, that:

$$z_3 < 2 \cdot (\omega\tau_0)^2 \tag{40}$$

For modeling substance in the equivalent $R\tau$-circuit of elementary volume of space at $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ it is necessary to use combinations of the circuits of a fig. 16, fig. 17, fig. 19 and fig. 20, depending on meanings ε_a and μ_a (see fig. 21 - 24).

The nominal values of elements for the circuits represented in a fig. 21 - 24 are calculated with the help of expressions (26), (28), (30), (35) and (37).

The constructed conservative $R\tau$-circuits of elementary volume of space for $\varepsilon_a \neq \varepsilon_0$ and $\mu_a \neq \mu_0$ planar (2D) task of H polarization, allow to receive calculated steady procedures of the electromagnetic analysis both in frequency and in time domain mode, since received at assemblage of the given circuits the grid of analyzed area too will be conservative [17].

CONCLUSION

The offered equivalent circuits of an element of space from the concentrated and distributed elements, allow to form conservative grids for 2D of the electromagnetic analysis of systems with arbitrary distribution permittivity and permeability in frequency and in time domain mode. The algorithms, constructed on the basis of such grids, of the analysis have numerical stability at meanings positive, negative and equal to zero, permittivity and permeability, that will be illustrated in the following publication.

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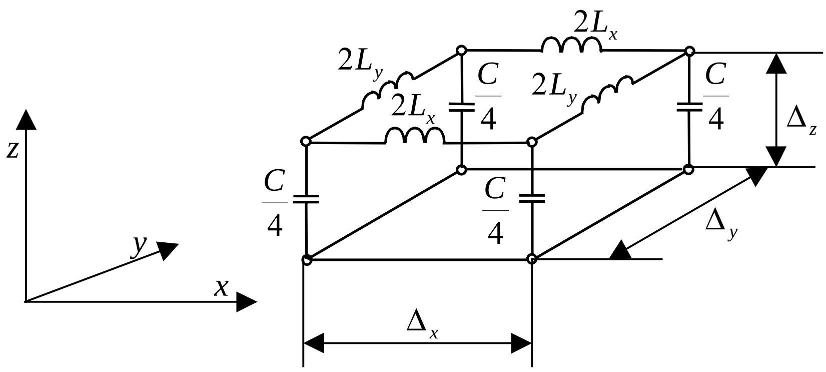

Fig. 1. The equivalent impedance circuit of an element of space.

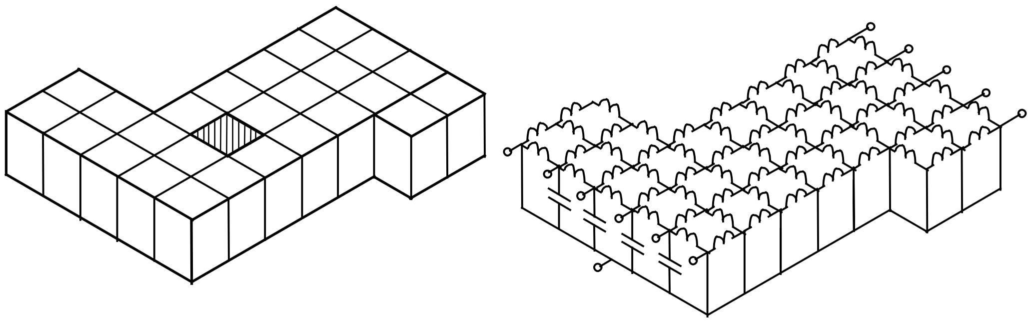

Fig. 2. Assemblage of the equivalent RLC -circuits of elementary volumes and formation of the equivalent circuit of all analyzed geometry.

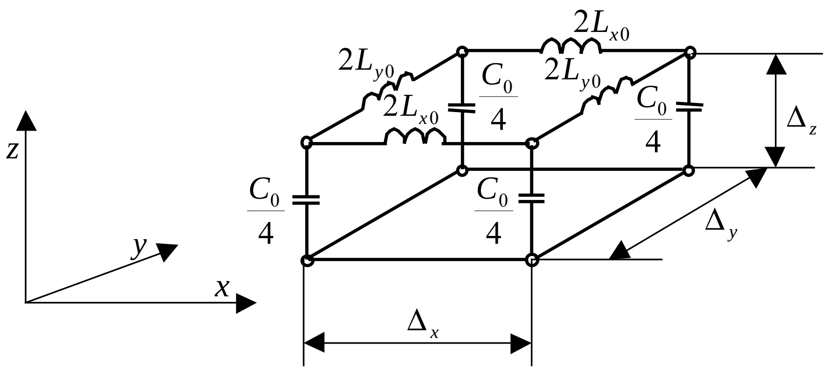

Fig. 3. The equivalent RLC -circuit of an element of vacuum.

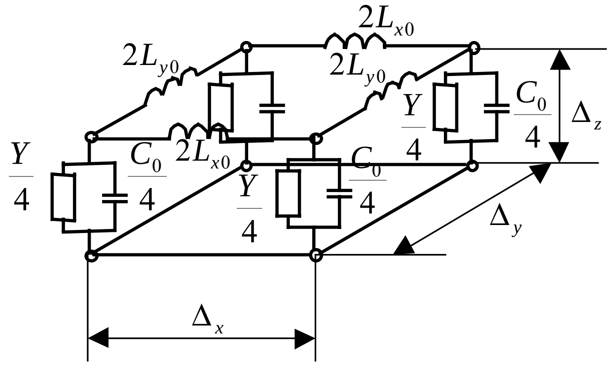

Fig. 4. Modeling in the RLC -circuit of an element of space of dielectric filling.

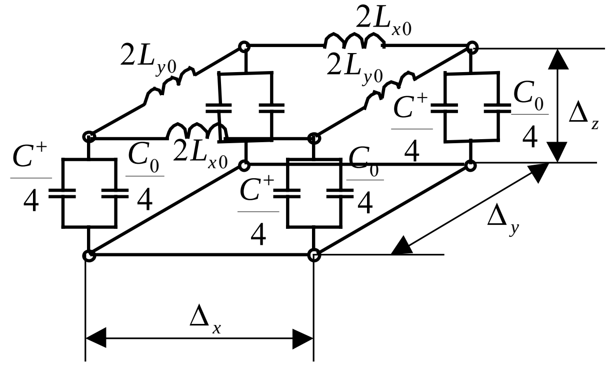

Fig. 5. Modeling in the RLC -circuit of an element of space of dielectric filling with substance $\varepsilon_a > \varepsilon_0$.

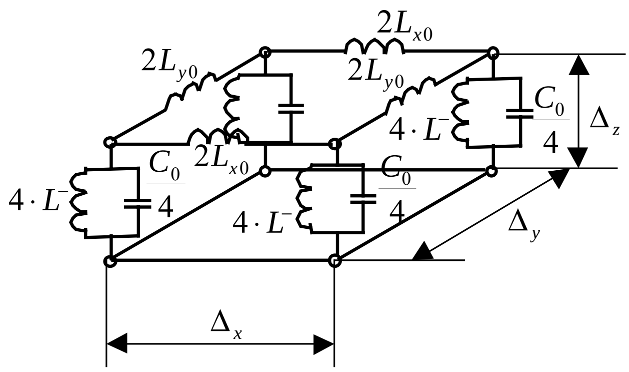

Fig. 6. Modeling in the RLC -circuit of an element of space of dielectric filling with substance $\varepsilon_a < \varepsilon_0$.

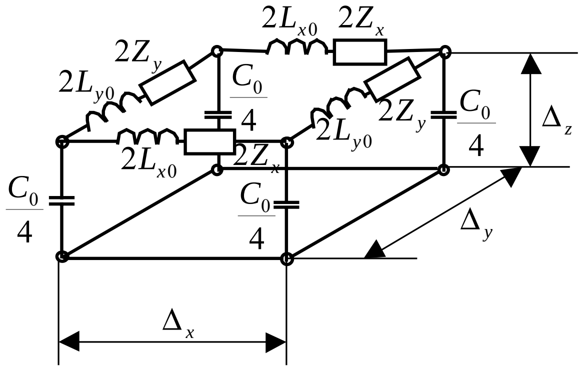

Fig. 7. Modeling in the RLC -circuit of an element of space of magnetic filling.

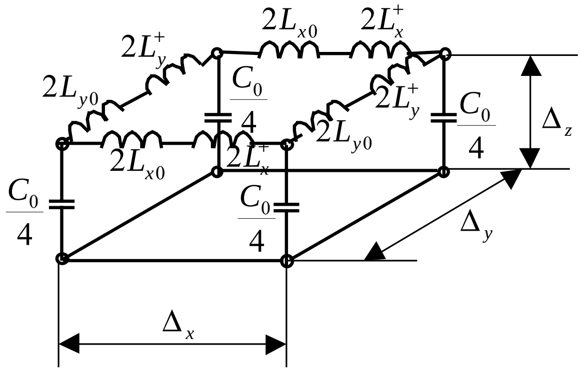

Fig. 8. Modeling in the RLC -circuit of an element of space of magnetic filling with substance $\mu_a > \mu_0$.

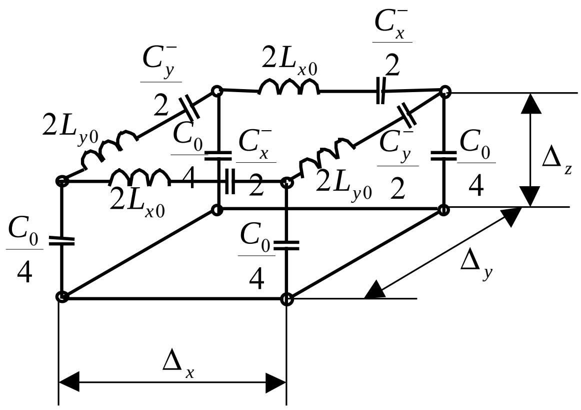

Fig. 9. Modeling in the RLC -circuit of an element of space of magnetic filling with substance $\mu_a < \mu_0$.

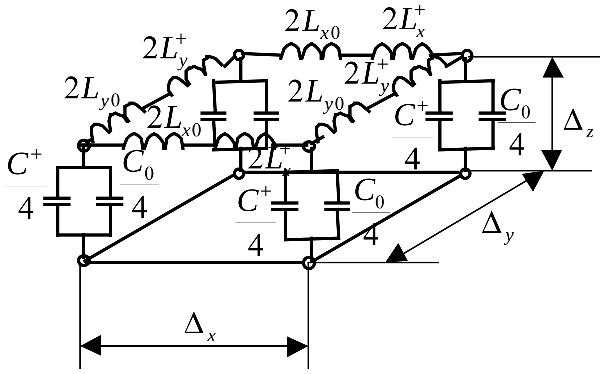

Fig. 10. Modeling in the RLC -circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a > \varepsilon_0$ and $\mu_a > \mu_0$.

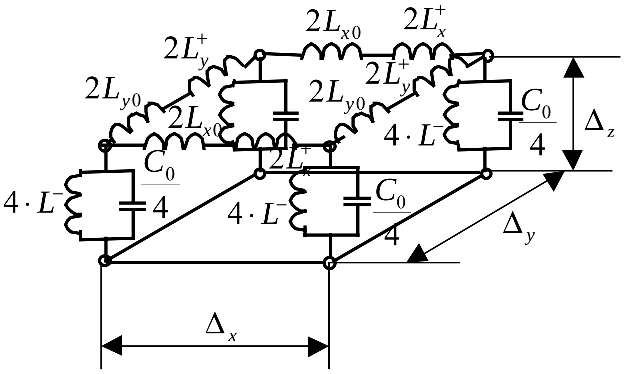

Fig. 11. Modeling in the RLC -circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a < \varepsilon_0$ and $\mu_a > \mu_0$.

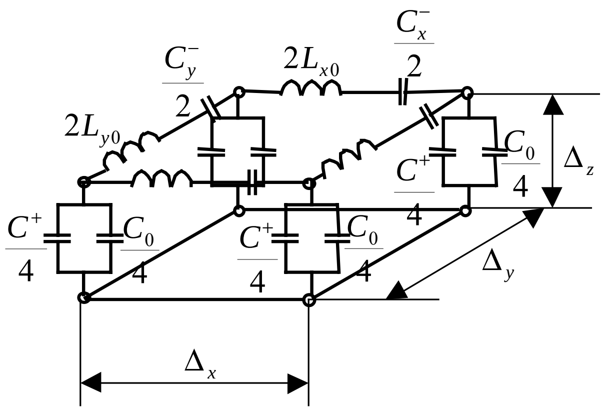

Fig. 12. Modeling in the RLC -circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a > \varepsilon_0$ and $\mu_a < \mu_0$.

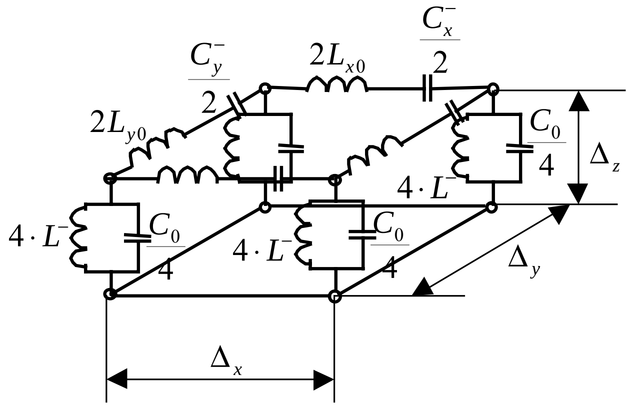

Fig. 13. Modeling in the RLC -circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a < \varepsilon_0$ and $\mu_a < \mu_0$.

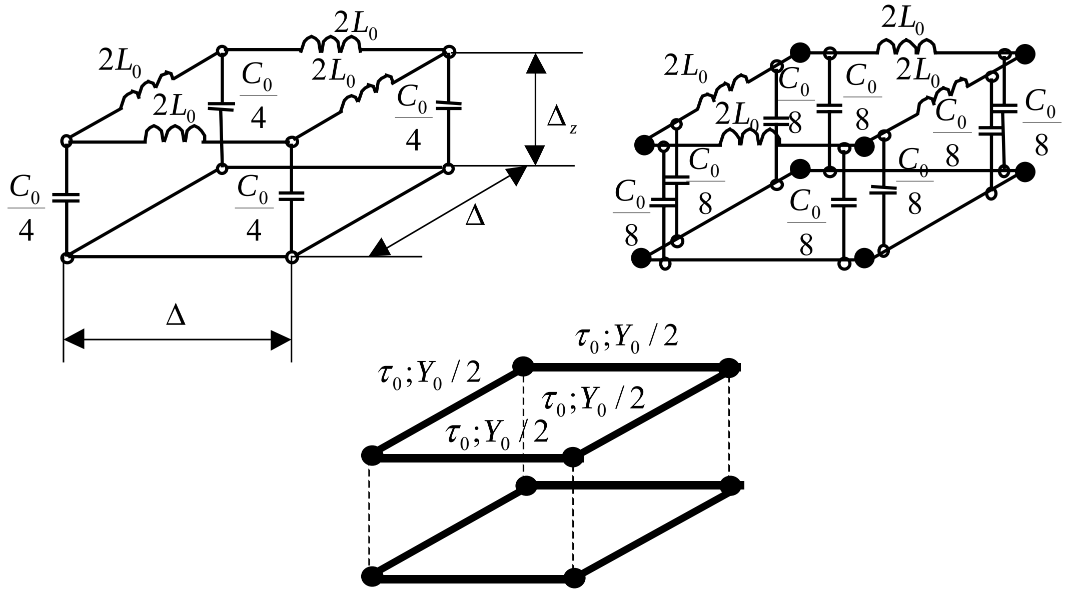

Fig. 14. Transformation of the RLC -equivalent circuit to the equivalent $R\tau$ -circuit for an element of space for vacuum.

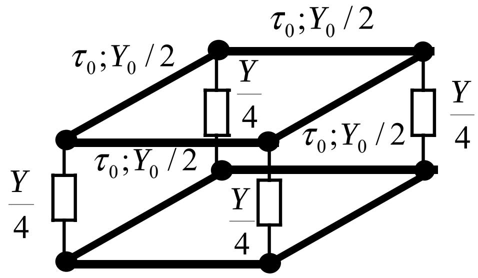

Fig. 15. Modeling in the $R\tau$ -circuit of an element of space of dielectric filling.

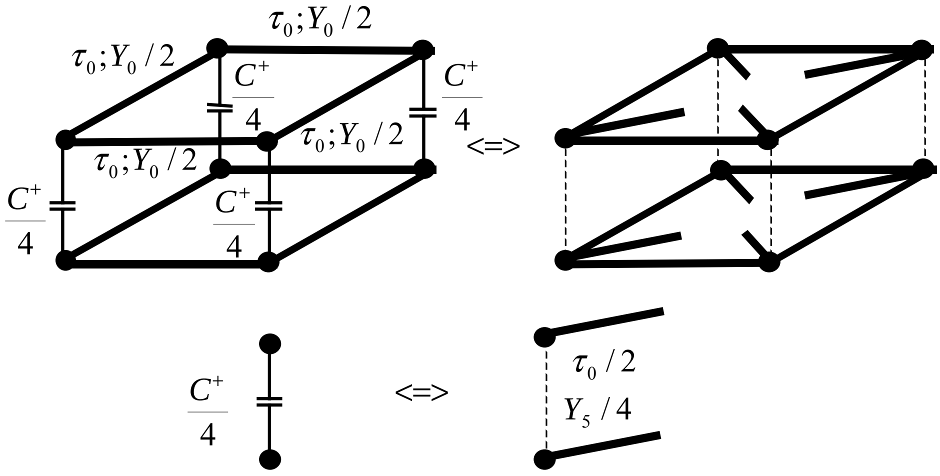

Fig. 16. Modeling in the $R\tau$ -circuit of an element of space of dielectric filling with substance $\varepsilon_a > \varepsilon_0$.

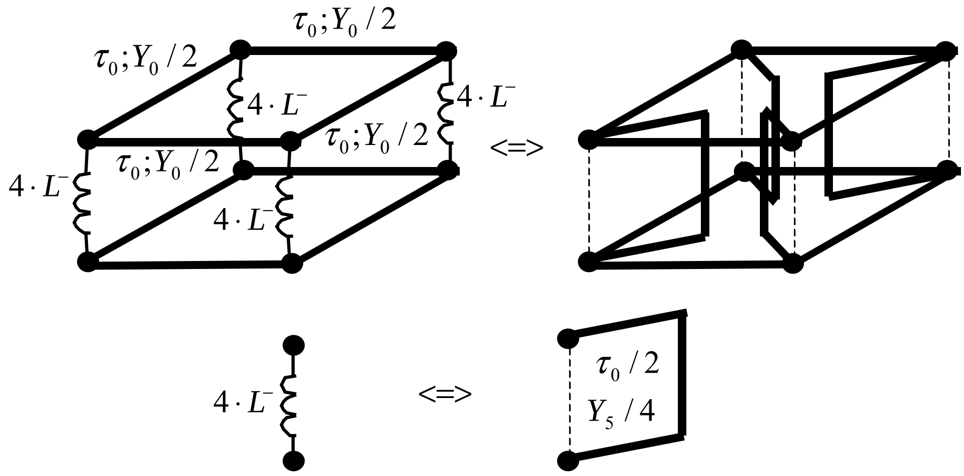

Fig. 17. Modeling in the $R\tau$ -circuit of an element of space of dielectric filling with substance $\varepsilon_a < \varepsilon_0$.

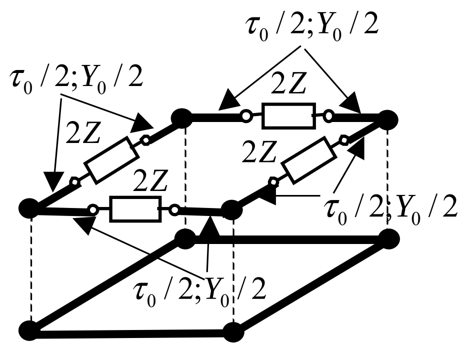

Fig. 18. Modeling in the $R\tau$ -circuit of an element of space of magnetic filling.

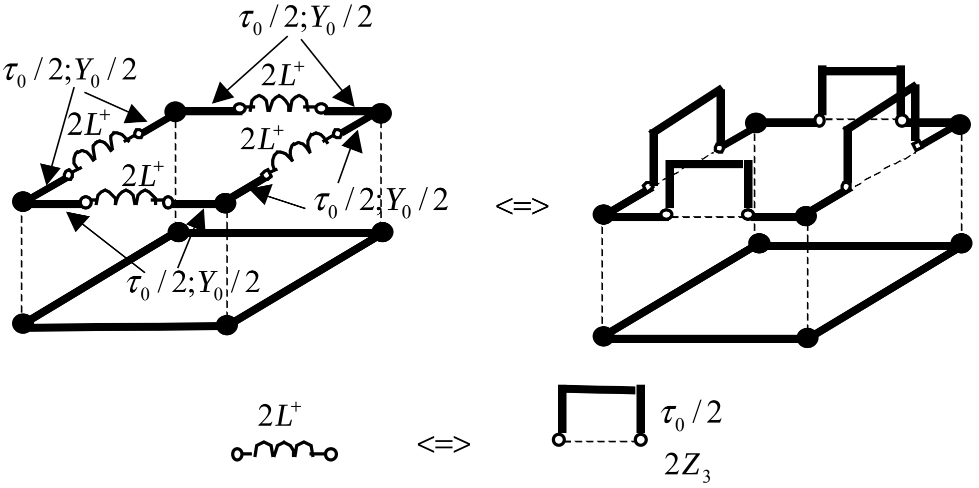

Fig. 19. Modeling in the $R\tau$ -circuit of an element of space of magnetic filling with substance $\mu_a > \mu_0$.

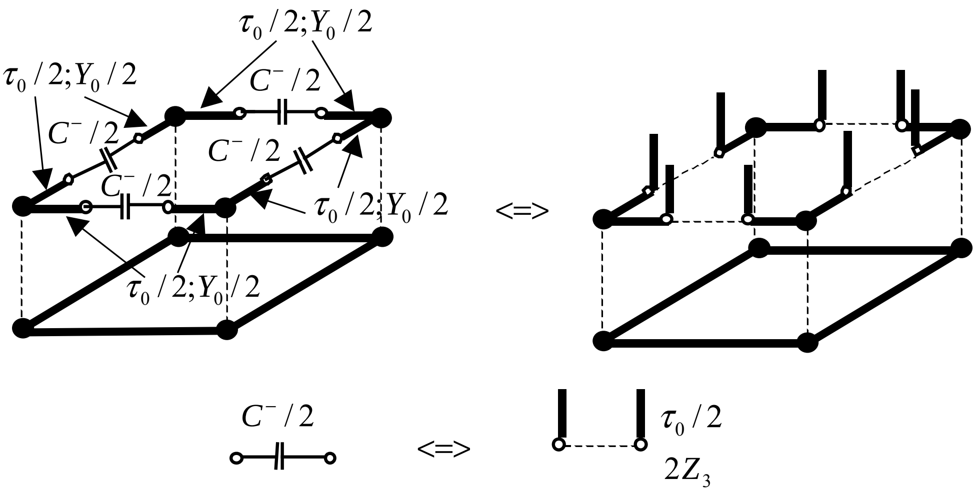

Fig. 20. Modeling in the $R\tau$ -circuit of an element of space of magnetic filling with substance $\mu_a < \mu_0$.

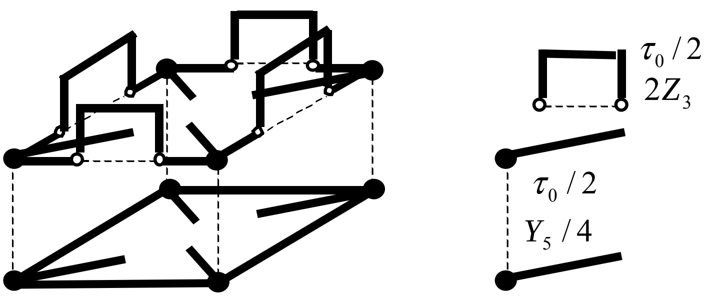

Fig. 21. Modeling in the $R\tau$ -circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a > \varepsilon_0$ and $\mu_a > \mu_0$.

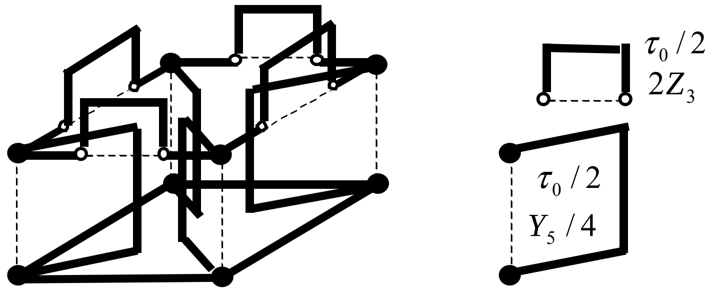

Fig. 22. Modeling in the $R\tau$-circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a < \varepsilon_0$ and $\mu_a > \mu_0$.

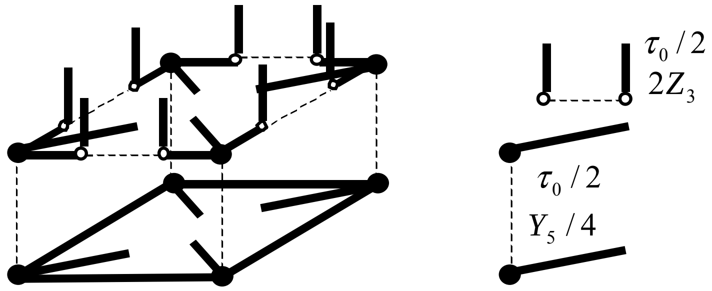

Fig. 23. Modeling in the $R\tau$-circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a > \varepsilon_0$ and $\mu_a < \mu_0$.

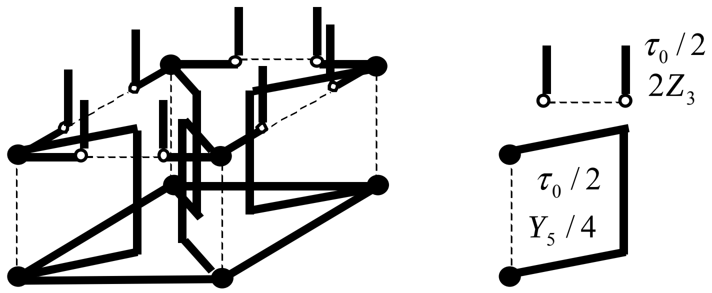

Fig. 24. Modeling in the $R\tau$-circuit of an element of space of dielectric and magnetic filling with substance $\varepsilon_a < \varepsilon_0$ and $\mu_a < \mu_0$.